where $H_n = S_n - (\bar{X} - \mu)(\bar{X} - \mu)^{\mathrm{T}}$. Thus, $q_n(\mu)$ can be readily computed without solving the nonlinear equation (5) as for the full empirical likelihood. The least-squares empirical likelihood ratio is a first-order approximation to the full empirical likelihood ratio, and $q_n(\mu) \to \chi_p^2$ in distribution when p is fixed.

The least-squares empirical likelihood is less affected by higher dimension. In particular, if $k \geqslant 3$ in (4), then

$$(2p)^{-1/2}\{q_n(\mu) - p\} \to N(0, 1) \tag{12}$$

in distribution as $n \to \infty$ when $p = o(n^{2/3})$, which improves the rate given by Theorem 3 for the full empirical likelihood ratio $w_n(\mu)$.

To appreciate (12), we note from (11) that

$$q_n(\mu) = n(\bar{X} - \mu)^{\mathrm{T}}\Sigma^{-1}(\bar{X} - \mu) + n(\bar{X} - \mu)^{\mathrm{T}}(H_n^{-1} - \Sigma^{-1})(\bar{X} - \mu). \tag{13}$$

Then, following a similar line to the proof of Lemma 6,

$$n(\bar{X} - \mu)^{\mathrm{T}}(H_n^{-1} - \Sigma^{-1})(\bar{X} - \mu) = O_p(p^2/n) = o_p(p^{1/2}).$$

As the first term on the right-hand side of (13) is asymptotically normal with mean p and variance $2p$ as conveyed in (10), (12) is valid.

If we confine ourselves to specific distributions, faster rates for p can be established. For example, if the data are normally distributed, the least-squares empirical likelihood ratio is the Hotelling-T^2 statistic, which is shown in Bai & Saranadasa (1996) to be asymptotically normal if $p/n \to c \in [0, 1)$.

$X = Z_1 + \rho Z_2$

4. NUMERICAL RESULTS

We report results from a simulation study designed to evaluate the asymptotic normality of the empirical likelihood ratio. The $p \times 1$ independent and identically distributed data vectors $\{X_i\}_{i=1}^n$ were generated from a moving average model,

$$X_{ij} = Z_{ij} + \rho Z_{ij+1} \quad (i = 1, \ldots, n, \quad j = 1, \ldots, p),$$

where, for each i, the innovations $\{Z_{ij}\}_{j=1}^{p+1}$ were independent random variables with zero mean and unit variance. We considered two distributions for the innovation Z_{ij}. One is the standard normal distribution, and the other is a standardized version of a Pareto distribution with distribution function $(1 - x^{-4\cdot5})I(x \geqslant 1)$. We standardized the Pareto random variables so that they had zero mean and unit variance. As the Pareto distribution has only four finite moments, we had $k = 1$ in (4), whereas $k = \infty$ for the normally distributed innovations. In both distributions, X_i is a multivariate random vector with zero mean and covariance $\Sigma = (\sigma_{ij})_{p\times p}$, where $\sigma_{ii} = 1, \sigma_{ii\pm1} = \rho$ and $\sigma_{ij} = 0$ for $|i - j| > 1$. We set ρ to be 0·5 throughout the simulation.

To make p and n increase simultaneously, we considered two growth rates for p with respect to n: (i) $p = c_1 n^{0\cdot4}$ and (ii) $p = c_2 n^{0\cdot24}$. We chose the sample size $n = 200$, 400 and 800. By assigning $c_1 = 3$, 4 and 5 in the faster growth rate setting (i), we obtained three dimensions for each sample size, which were $p = 25$, 33 and 43 for $n = 200$; $p = 33$, 44 and 58 for $n = 400$; and $p = 42$, 55 and 72 for $n = 800$, respectively. For the slower growth rate setting (ii), to maintain a certain amount of increase between successive dimensions when n was increased, we assigned larger $c_2 = 4$, 6 and 8, which led to $p = 14$, 17 and 20 for $n = 200$; $p = 21$, 25 and 30 for $n = 400$; and $p = 29$, 34 and 40 for $n = 800$, respectively.

We carried out 500 simulations for each of the (p, n)-combinations and for each of the two innovation distributions. Figure 1 displays Q-Q plots of standardized empirical likelihood ratio

This content downloaded from 116.1.3.20 on Mon, 24 May 2021 08:40:48 UTC
All use subject to https://about.jstor.org/terms