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Empirical likelihood inference for partially linear panel data models with fixed effects

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ABSTRACT

We propose an empirical likelihood method for application to a partially linear panel data model with fixed effects. The empirical log-likelihood ratio statistic is proved to be asymptotically chi-squared distributed, and the asymptotic properties of estimators for both the parametric and nonparametric components are established.

1. Introduction

With the increasing availability of panel data, both theoretical and applied works in panel data analysis have become more popular in recent years. Baltagi (2005) and Hsiao (2003) provide an excellent overview of parametric panel data analysis. To avoid imposing the strong restrictions assumed in the parametric panel data models, nonparametric/semiparametric panel data models have recently received much attention, including that from Henderson et al. (2008), Sun et al. (2009), Baltagi and Li (2002), Li and Stengos (1996) and Su and Ullah (2006), among others.

Consider the following partially linear panel data models with fixed effects:

$$Y_{it} = \mu_i + X_{it}^{\tau}\beta + g(Z_{it}) + V_{it}, \quad i = 1, \dots, n, \ t = 1, \dots, T, \quad (1.1)$$

where X_{it} and Z_{it} are of dimensions $p \times 1$ and $q \times 1$ respectively, β is a $p \times 1$ vector of unknown parameters, $g(\cdot)$ is an unknown smooth function, μ_i $(i = 1, \dots, n)$ are fixed effects, and V_{it} is the random model error. For simplicity, we assume that (X_{it}, Z_{it}) are strictly exogenous variables. The random errors V_{it} are assumed to be i.i.d. with zero mean and finite variance, and independent of μ_k, X_{ks}, and Z_{ks} for all i, k, s and t. The unobserved individual effects μ_i are assumed to be i.i.d. with zero mean and finite variance $\sigma_u^2 > 0$. We allow μ_i to be correlated with X_{it} and Z_{it} with an unknown correlation structure. Hence, model (1.1) is a fixed effects model. As a special case, when μ_i is uncorrelated with X_{it} and Z_{it}, model (1.1) becomes a random effects model.

Although many studies have focused on consistent estimation of semiparametric panel data models with fixed effects, to our knowledge, none has considered the empirical likelihood (EL) method proposed by Owen (1988) in a dynamic partially linear panel data model with fixed effects. It is well known that there are some striking advantages for the EL method in the construction of confidence regions for unknown parameters. For example, the EL-based inferences do not involve covariance estimation and the EL method determines the shape and orientation of confidence regions based on data. The EL method has been successfully applied to various statistical models, including for example those of Qin and Lawless (1994), Cui and Kong (2006), Zhu and Xue (2006) and Li et al. (2010), among others.

The purpose of this work is to apply the EL method to construct an empirical log-likelihood ratio (ELR) statistic for the parameter vector β, and to prove that the proposed statistic has asymptotic

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chi-squared distribution. To obtain accurate confidence regions, an important step is to remove the fixed individual effects from the model. The proposed approach draws support from Wang (2003), Su and Ullah (2006) and Lin and Carroll (2006) on profile likelihood, which is extremely useful for estimating semiparametric models. Given the finite dimensional parameter of interest and the fixed effect parameters, we can estimate the nonparametric part as a function of these parameters. Then we obtain the ELR statistic by plugging this nonparametric component into an auxiliary random vector. In addition, by introducing the kernel-based weights, the fixed effects are removed and an ELR statistic for the unknown parameter of interest is suggested. We establish the asymptotic theories for the parametric and nonparametric components by letting n approach infinity while holding T fixed. The proposed method also provides estimates for the fixed effects.

The work is organized as follows. In Section 2, we propose the procedure for removing the fixed individual effects, suggest the ELR statistic of the unknown parameters and give the estimators of the parametric and nonparametric components. Some asymptotic properties are established in Section 3. Concluding remarks are presented in Section 4. The simulation studies and the technical details are available upon request, and they are relegated to supplementary material.

2. Methodology

Let $K(\cdot)$ denote a kernel function on $\mathbb{R}^q$, and let $H = \mathrm{diag}(h_1, \ldots, h_q)$ be the $q \times q$ diagonal bandwidth matrix. Set $K_H(z) = |H|^{-1}K(H^{-1}z)$, where $|H|$ is the determinant of H. If Z_{it} is in a small neighborhood of z, we can approximate $g(Z_{it})$ locally by a linear function as follows:

$$g(Z_{it}) \approx g(z) + \{Hg'(z)\}^\tau [H^{-1}(Z_{it} - z)], \tag{2.1}$$

where $g'(z) = \partial g(z)/\partial z$ is the $q \times 1$ vector of the first-order partial derivatives.

We will use vector and matrix notation in the following. Let I_T denote an identity matrix of dimension T, and e_T denote a $T \times 1$ vector with all elements being ones. $\mathbf{Y}_i = (Y_{i1}, \ldots, Y_{iT})^\tau$ and $\mathbf{V}_i = (V_{i1}, \ldots, V_{iT})^\tau$ are $T \times 1$ vectors, and $\mathbf{X}_i = (X_{i1}, \ldots, X_{iT})^\tau$ and $\mathbf{Z}_i = (Z_{i1}, \ldots, Z_{iT})^\tau$ are $T \times p$ and $T \times q$ matrices, respectively. Define a $T \times T$ diagonal matrix $\mathbf{K}_H(z) = \mathrm{diag}\{K_H(Z_{i1} - z), \ldots, K_H(Z_{iT} - z)\}$ for each i, where $K_H(Z_{it} - z) = |H|^{-1}K\{H^{-1}(Z_{it} - z)\}$ for all i and t. Let $B_{it}(z, H) = [1, \{H^{-1}(Z_{it} - z)\}^\tau]^\tau$ be a $(q + 1) \times 1$ vector, $B_i(z, H) = (B_{i1}(z, H), \ldots, B_{iT}(z, H))^\tau$ be a $T \times (q + 1)$ matrix, and $\mathbf{G}(z) = (g(z), (Hg'(z))^\tau)^\tau$ be a $(q + 1) \times 1$ vector.

Given μ_i $(i = 1, \ldots, n)$ and β, $\mathbf{G}(z) = (g(z), (Hg'(z))^\tau)^\tau$ can be estimated by minimizing

$$\min_{\mathbf{G}\in\mathbb{R}^{q+1}} \sum_{i=1}^{n} (\mathbf{Y}_i - e_T\mu_i - \mathbf{X}_i\beta - B_i(z, H)\mathbf{G}(z))^\tau \mathbf{K}_H(z) \times (\mathbf{Y}_i - e_T\mu_i - \mathbf{X}_i\beta - B_i(z, H)\mathbf{G}(z)). \tag{2.2}$$

Define the smoothing operator

$$S(z) = \left(\sum_{i=1}^{n} B_i^\tau(z, H)\mathbf{K}_H(z)B_i(z, H)\right)^{-1} \sum_{i=1}^{n} B_i^\tau(z, H)\mathbf{K}_H(z).$$

Suppose that $\mu = (\mu_1, \ldots, \mu_n)^\tau$ and $\gamma = (\mu^\tau, \beta^\tau)^\tau$. Then, the estimator for $g(z)$ is given by

$$\hat{g}(z, \gamma) = s^\tau(z)(\mathbf{Y}_i - e_T\mu_i - \mathbf{X}_i\beta), \tag{2.3}$$

where $s^\tau(z) = \mathrm{e}^\tau S(z)$, and $e = (1, 0, \ldots, 0)^\tau$ is a $(q+1) \times 1$ vector. Now we consider a way of removing the unknown fixed effects motivated by a least squares dummy variable model in parametric panel data analysis, for which we solve the following optimization problem:

$$\min_{\mu} \sum_{i=1}^{n} (\mathbf{Y}_i - e_T\mu_i - \mathbf{X}_i\beta - \hat{g}(\mathbf{Z}_i, \gamma))^\tau \times (\mathbf{Y}_i - e_T\mu_i - \mathbf{X}_i\beta - \hat{g}(\mathbf{Z}_i, \gamma)), \tag{2.4}$$

where $\hat{g}(\mathbf{Z}_i, \gamma) = (\hat{g}(Z_{i1}, \gamma), \ldots, \hat{g}(Z_{iT}, \gamma))^\tau$. Substituting (2.3) into (2.4), taking partial derivatives with respect to μ_i and setting them equal to zero, we have

$$\hat{\mu}_i(\beta) = (e_T^\tau(I_T - S_i)^\tau(I_T - S_i)e_T)^{-1} \times e_T^\tau(I_T - S_i)^\tau(I_T - S_i)(\mathbf{Y}_i - \mathbf{X}_i\beta), \tag{2.5}$$

where $S_i = (s(Z_{i1}), \ldots, s(Z_{iT}))^\tau$ is a $T \times T$ smoothing matrix. Plugging (2.3) and (2.5) into (2.4) and supposing that $\widetilde{\mathbf{X}}_i = (I_T - S_i)\mathbf{X}_i$, $\widetilde{\mathbf{Y}}_i = (I_T - S_i)\mathbf{Y}_i$, $\widetilde{e}_T = (I_T - S_i)e_T$, and $\widetilde{M}_i = I_T - \widetilde{e}_T(\widetilde{e}_T^\tau\widetilde{e}_T)^{-1}\widetilde{e}_T^\tau$, we introduce the following auxiliary random vector:

$$\eta_i(\beta) = \widetilde{\mathbf{X}}_i^\tau \widetilde{M}_i(\widetilde{\mathbf{Y}}_i - \widetilde{\mathbf{X}}_i\beta). \tag{2.6}$$

Note that $E(\eta_i(\beta)) = 0$ if β is the true parameter. Therefore, using the information $E(\eta_i(\beta)) = 0$, the empirical log-likelihood ratio (ELR) is defined as

$$l(\beta) = \max\left\{\sum_{i=1}^{n} \log(np_i) | p_i \geq 0,\ i = 1, \ldots, n,\ \sum_{i=1}^{n} p_i = 1, \sum_{i=1}^{n} p_i\eta_i(\beta) = 0\right\}. \tag{2.7}$$

Using the Lagrange multiplier method, the optimal value for p_i is given by

$$p_i = \frac{1}{n}\frac{1}{1 + \lambda^\tau(\beta)\eta_i(\beta)}, \quad i = 1, \ldots, n. \tag{2.8}$$

By (2.7) and (2.8), $l(\beta)$ can be represented as

$$l(\beta) = -\sum_{i=1}^{n} \log(1 + \lambda^\tau(\beta)\eta_i(\beta)), \tag{2.9}$$

where $\lambda(\beta)$ is the Lagrange multiplier, and is determined by

$$\frac{1}{n}\sum_{i=1}^{n} \frac{\eta_i(\beta)}{1 + \lambda^\tau(\beta)\eta_i(\beta)} = 0. \tag{2.10}$$

We can obtain the maximum empirical likelihood estimator (MELE) $\hat{\beta}$ of β by maximizing $l(\beta)$. According to Qin and Lawless (1994), $\hat{\beta}$ is also equal to the solution of the estimating equations $\sum_{i=1}^{n} \eta_i(\beta) = 0$ given by

$$\widetilde{\beta} = \left(\sum_{i=1}^{n} \widetilde{\mathbf{X}}_i^\tau \widetilde{M}_i \widetilde{\mathbf{X}}_i\right)^{-1} \sum_{i=1}^{n} \widetilde{\mathbf{X}}_i^\tau \widetilde{M}_i \widetilde{\mathbf{Y}}_i, \tag{2.11}$$

which is called the profile least squares estimator (PLSE) (see Su and Ullah, 2006). Therefore, the MELE for β is identical to the PLSE. Furthermore, we can obtain estimates of μ_i $(i = 1, \ldots, n)$ and $g(\cdot)$, say $\hat{g}_n(z)$, as follows:

$$\hat{\mu}_i = (\widetilde{e}_T^\tau\widetilde{e}_T)^{-1}\widetilde{e}_T^\tau(\widetilde{\mathbf{Y}}_i - \widetilde{\mathbf{X}}_i\hat{\beta}), \quad i = 1, \ldots, n \tag{2.12}$$

and

$$\hat{g}_n(z) = s^\tau(z)(\mathbf{Y}_i - e_T\hat{\mu}_i - \mathbf{X}_i\hat{\beta}). \tag{2.13}$$

3. Asymptotic properties

To present the asymptotic properties in this section, we first list the following regularity conditions. We then present the asymptotic distribution of $-2l(\beta)$ and the asymptotic normality of the resulting estimators.

C1. $E\|X_{it}\|^4 < \infty$ and $\sup_{1\leq i\leq n} E\|\mathbf{V}_{it}\|^4 < \infty$. Suppose that $\sigma^2(x, z) = \mathrm{Var}(Y_{it}|X_{it} = x, Z_{it} = z)$ and $\sigma^2(z) = \mathrm{Var}(Y_{it}|Z_{it} = z)$.

$\sigma^2(x, z)$ and $\sigma^2(z)$ are uniformly bounded from infinity and zero. The error $\mathbf{V}_i = (V_{i1}, \ldots, V_{iT})^\tau$ has a positive definite covariance matrix $\Omega_i = E(\mathbf{V}_i\mathbf{V}_i^\tau)$.

C2. $E(Y_{it}|\mathbf{X}_i, \mathbf{Z}_i, \mu_i) = E(Y_{it}|X_{it}, Z_{it}, \mu_i) = \mu_i + X_{it}^\tau\beta + g(Z_{it})$.

C3. Z_{it} has a continuous density function $f_t(\cdot)$ with compact support C_f on $\mathbb{R}^q$. $f_t(\cdot)$ is bounded away from zero and infinity on C_f for each $t = 1, \ldots, T$.

C4. Suppose that $p(z) = E(X_{it}|Z_{it} = z)$. The functions $g(\cdot)$ and $p(\cdot)$ have bounded second partial derivatives on C_f.

C5. Suppose that $\overline{X}_{it} = X_{it} - E(X_{it}|Z_{it})$. $\Phi = \sum_{t=1}^T E\{\overline{X}_{it}[\overline{X}_{it} - \sum_{s=1}^T \overline{X}_{is}/T]^\tau\}$ is positive definite.

C6. The kernel function $K(\cdot)$ is a continuous density with compact support on $\mathbb{R}^q$. All odd order moments of K vanish.

C7. Let $|H| = h_1 \cdots h_q$ be the determinant of H and suppose that $\|H\| = \sqrt{\sum_{j=1}^q h_j^2}$. As $n \to \infty$, $\|H\| \to 0$, $n|H|^2 \to \infty$, $\|H\|^4 |H|^{-1} \to 0$ and $n|H|\|H\|^4 \to c \in [0, \infty)$.

These conditions are quite mild and similar conditions can be found in Su and Ullah (2006) and Sun et al. (2009). In order to study the asymptotic properties, suppose that $f(z) = \sum_{t=1}^T f_t(z)$, $\overline{V}_{it} = V_{it} - \frac{1}{T}\sum_{s=1}^T V_{is}$, $\sigma_t^2(z) = E[\overline{V}_{it}^2|Z_{it} = z]$, and $\overline{\sigma}^2(z) = \sum_{t=1}^T \sigma_t^2(z)f_t(z)$.

Theorem 1. *Suppose Conditions* C1–C7 *hold. If β_0 is the true value of the parameter β, then $-2l(\beta_0)$ is asymptotically chi-squared distributed with p degrees of freedom as $n \to \infty$.*

As a consequence of this result, confidence regions for the parameter β can be constructed. For any given $0 < \alpha < 1$, there exists c_α such that $P(\chi_p^2 > c_\alpha) = \alpha$; then

$$I_\alpha(\beta) = \{\beta \in \mathbb{R}^p \mid -2l(\beta) \le c_\alpha\}$$

constitutes a confidence region for β with asymptotic coverage $1 - \alpha$.

The following theorems state the asymptotic behavior of the MELE and the estimator for the nonparametric component proposed in Section 2, respectively.

Theorem 2. *Suppose Conditions* C1–C7 *hold. Then as $n \to \infty$, we have*

$$\sqrt{n}(\hat{\beta} - \beta) \xrightarrow{L} N(0, \Phi^{-1}\Lambda\Phi^{-1}),$$

where "$\xrightarrow{L}$" denotes the convergence in distribution, Φ is defined in condition C5, *and $\Lambda = E(\overline{\mathbf{X}}_i^\tau \Omega_i \overline{\mathbf{X}}_i)$ with $\overline{\mathbf{X}}_i = (\overline{X}_{i1}, \ldots, \overline{X}_{iT})^T$.*

To make inferences on β using Theorem 2, a plug-in estimator of the limiting variance of $\hat{\beta}$ is needed. The consistent estimator of $\Phi^{-1}\Lambda\Phi^{-1}$ is given by $\hat{\Phi}^{-1}\hat{\Lambda}\hat{\Phi}^{-1}$, where $\hat{\Phi} = (nT)^{-1}\sum_{i=1}^n \sum_{t=1}^T \hat{X}_{it}(\hat{X}_{it} - \sum_{l=1}^T \hat{X}_{il}/T)^\tau$, $\hat{\Lambda} = n^{-1}\sum_{i=1}^n \hat{\mathbf{X}}_i^\tau \hat{\mathbf{V}}_i \hat{\mathbf{V}}_i^\tau \hat{\mathbf{X}}_i$, $\hat{\mathbf{X}}_i = (\hat{X}_{i1}, \ldots, \hat{X}_{iT})^\tau$, $\hat{X}_{it}^\tau = X_{it}^\tau - s^\tau(Z_{it})X$, and $\hat{V}_{it} = Y_{it} - X_{it}^\tau\hat{\beta} - \hat{g}_n(Z_{it}) - \hat{\mu}_i$.

Theorem 3. *Suppose Conditions* C1–C7 *hold. Then as $n \to \infty$, we have*

$$\sqrt{n|H|}\left(\hat{g}_n(z) - g(z) - b(z)\right) \xrightarrow{L} N(0, \Sigma_{g(z)}),$$

where $b(z) = \frac{1}{2}\mathrm{tr}\left(\int_{\mathbb{R}^q} uu^\tau K(u)\mathrm{d}u(Hg''(z)H)\right)$ and $\Sigma_{g(z)} = f^{-2}(z)\,\overline{\sigma}^2(z)\int_{\mathbb{R}^q} K^2(u)\mathrm{d}u$.

From Theorem 3, it is easy to see that the resulting nonparametric estimator has the same asymptotic distribution as that in Su and Ullah (2006). Theorem 3 can be used to construct the pointwise confidence band for $g(z)$.

4. Concluding remarks

This article considers the empirical likelihood inferences for the partially linear panel data model with fixed effects. By using a local linear regression approach and the kernel-based weights, the fixed effects are removed and the ELR statistic for the unknown parameters of interest in the model is suggested. It is shown that asymptotically the proposed ELR statistic has a chi-squared distribution under some suitable conditions, and hence it can be used to construct the confidence region of the parameters. In addition, we also establish the asymptotic normality of the resulting MELE for the parametric component and the estimator for the nonparametric component. In the supplementary material, a modified "leave-one-subject-out" cross-validation method is used to select the optimal bandwidth automatically, and a simulation study is provided to assess the performance of the proposed method and compare it with the PLS method. The simulation study indicates that, in terms of coverage probabilities of the confidence regions, the proposed method performs better than the PLS method. The methods described here can be easily extended to various panel data models with fixed effects. These and other extensions are the subject of ongoing research.

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